647510 **BSc4MatC0401** **Seat No :________**

B.Sc. Semester - 4 (*CBCS*) Examination
March/April-2024 [OLD COURSE]
MATHEMATICS(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one of the following questions. (07)

(1) Let $V = \{(x, y) \in \mathbb{R}^2 : y > 0\}$. For $(a, b), (c, d) \in V$ and $\alpha \in \mathbb{R}$, define $(a, b) + (c, d) = (a + c, bd)$ and $\alpha(a, b) = (\alpha a, b^\alpha)$. Show that $(V, +, \cdot)$ is a vector space over $\mathbb{R}$.

(2) Let V be a vector space and $S = \{v_1, v_2, . \ ., v_n\} \subset V$ such that $sp\, S = V$. If $w_1, w_2, \ . \ . \ . \ , w_m$ are linearly independent vectors of V then prove that $m \leq n$.

Que.1(B) Answer any one of the following questions. (04)

(1) Define linear span of a subset of a vector space V and show that it is a subspace of V.

(2) Justify: A set containing zero vector is always linearly dependent.

Que.1(C) Answer any one of the following questions. (03)

(1) Check whether the set $S = \{(1,2,-1), (-1, 0, 1), (0,1,2)\}$ is linearly independent or not.

(2) Check whether $W = \{ (x, y, z) \in \mathbb{R}^3 : xy = 0\}$ is a subspace of $\mathbb{R}^3$or not.

Que.2(A) Answer any one of the following questions. (07)

(1) Prove that every linearly independent set in a vector space V can be extended to form a basis of V.

(2) Find $\dim(W_1 + W_2)$, where $W_1 = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_3 - x_4 = 0\}$ and $W_2 = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + 2x_2 = 0\}$ are subspaces of $\mathbb{R}^4$.

Que.2(B) Answer any one of the following questions. (04)

(1) Check whether the set $B = \{2, x, x^2 - x, x^3 + 2x^2 - 3x\}$ is a base of vector space $P_3(\mathbb{R})$ or not.

(2) Find co-ordinates of the vector $v = (-1, 2, 3)$ with respect to the basis $B = \{(-1,1,1), (0, -1,1), (0,0,1)\}$ of $\mathbb{R}^3$.

Que.2(C) Answer any one of the following questions. (03)

(1) Show that distinct basis of a vector space contains same number of elements.

(2) Define Basis and dimension of a vector space.

Que.3(A) Answer any one of the following questions. (07)

(1) State and prove rank nullity theorem.

(2) In usual notations find N_T, R_T, n_T, r_T for the linear map $T: \mathbb{R}^4 \to \mathbb{R}^3$ defined by

$$T\big((x_1, x_2, x_3, x_4)\big) = (\ x_1 + x_2, x_2 - x_3, x_3 + x_4), \forall\ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4.$$

Que.3(B) Answer any one of the following questions. (04)

(1) Find a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that

$$T(e_1) = (1,1,1),\ T(e_1 + e_2) = (1,1,0) \text{ and } T(e_1 + e_2 + e_3) = (1,0,0).$$

(2) Let $T: U \to U$ be a linear transformation. Prove that T is one-one $\Leftrightarrow$ T is onto.

Que.3(C) Answer any one of the following questions. (03)

(1) Check whether $T: \mathbb{R}^2 \to \mathbb{R}$ defined as $T\big((x_1, x_2)\big) = 2x_1 + 3x_2, \forall\ (x_1, x_2) \in \mathbb{R}^2$ is linear map or not.

(2) If $T: U \to V$ is a linear transformation then in usual notations prove that $T(\theta) = \theta$ and $T(-u) = -T(u),\ \forall\ u \in U$.

Que.4(A) Answer any one of the following questions. (07)

(1) Find $[T : B_1, B_2]$ for the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^4$ defined as

$$T\big((x_1, x_2, x_3)\big) = (\ x_1, -2x_2, x_2 + \ 3x_3, x_3 - x_1), \forall\ (x_1, x_2, x_3) \in \mathbb{R}^3.$$

where B_1 and B_2 are standard basis of $\mathbb{R}^3$ and $\mathbb{R}^4$, respectively.

(2) Find eigen values and eigen vectors of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T\big((x_1, x_2)\big) = (\ x_1 + x_2,\ 2x_2), \forall\ (x_1, x_2) \in \mathbb{R}^3$.

Que.4(B) Answer any one of the following questions. (04)

(1) Let $T: V \to V$ be a linear transformation such that $T^2 - T + I = 0$, then prove that T is non-singular.

(2) Let $T: V \to V$ be a linear transformation and let B be a basis of V. Prove that T is singular iff $\det([T; B]) = 0$.

Que.4(C) Answer any one of the following questions. (03)

(1) Define dual of a vector space and adjoint of a linear transformation.

(2) Define eigen value and eigen vector of a linear transformation.

Que.5(A) Answer any one of the following questions. (07)

(1) Find all asymptotes to the curve $xy^2 + y^3 = x + y + 1$.

(2) Find multiple points of the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$ and determine their types.

Que.5(B) Answer any one of the following questions. (04)

(1) Find radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$.

(2) Find point of inflexion for the curve $y^3 + 3y^2 - x = 0$.

Que.5(C) Answer any one of the following questions. (03)

(1) Prove that the curve $y = \log x$ is convex upwards on R^+.

(2) Find radius of curvature at the origin for the curve $x^2 + 2xy + 2y^2 - 4x = 0$.

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